**HW #7**  
*Due 30 November 2015 in class*

**Problem 1**  
The flow at the entrance to an axial-flow compressor rotor has zero preswirl and an axial velocity of 175 m/s. The shaft angular speed is 5,000 rpm. If at a radius of 0.5 m, the rotor exit flow has zero relative swirl, calculate at this radius:

(a) rotor specific work in kJ/kg  
(b) degree of reaction, °R

**Problem 2**  
Consider a rotor blade row at the pitchline \( r_m \). Suppose that the rotor blade chord is \( c = 6 \) cm and the blade-to-blade spacing is \( s = 5 \) cm. Assuming that the rotor linear velocity is 210 m/s, that \( C_{z_1} = C_{z_2} = 175 \) m/s, \( \rho_1 = 1 \text{ g/m}^3 \), \( \beta_2 = -25^\circ \), and \( \sigma = 0.03 \), calculate

(a) The relative swirl velocities, \( W_{\theta_1} \) and \( W_{\theta_2} \)  
(b) The relative velocity magnitudes, \( W_1 \) and \( W_2 \)  
(c) The diffusion factor, \( D \)  
(d) The circulation, \( \Gamma \)  
(e) The rotor lift per unit span, \( L' \)  
(f) The lift coefficient, \( C_\ell \)  
(g) The rotor specific work, \( w_c \)  
(h) The loading coefficient, \( \Psi \)  
(i) The degree of reaction, °R.

**Problem 3**  
Consider a stator where the absolute flow angle into the stator is \( \alpha_2 = 45^\circ \), with total pressure and temperature \( p_{t_2} = 150 \text{ kPa} \), \( T_{t_2} = 300 \text{ K} \), respectively. The total pressure loss coefficient is \( \sigma = 0.02 \). Assuming the axial velocity remains constant and gas properties are \( \gamma = 1.4 \) and \( C_p = 1.004 \text{ kJ/(kg·K)} \), calculate:

(a) Entrance Mach number, \( M_2 \)  
(b) Exit total pressure, \( p_{t_3} \)  
(c) Exit Mach number, \( M_3 \)  
(d) Stator torque for a mass flow rate of \( \dot{m} = 100 \text{ kg/s} \)  
(e) Static pressure rise, \( p_3 - p_2 \)  
(f) Static temperature rise, \( T_3 - T_2 \)  
(g) Entropy rise, \( (s_3 - s_2)/R \).
Problem 4  When discussing the compressor blades we introduced the idea of a total pressure loss coefficient, $\sigma$. When $\sigma > 0$ it implies there are losses due to non-isentropic processes associated with the compressor. This further implies that the compressor as a whole has an efficiency that is not equal to one. It is typical for a compressor to be described by two different efficiency measures such that $\tau_c \neq \pi_c^{(\gamma-1)/\gamma}$. The first efficiency is the compressor adiabatic efficiency, $\eta_c$, defined by

$$\eta_c = \frac{\text{isentropic } \Delta h_t}{\text{actual } \Delta h_t} = \frac{h_{3s} - h_2}{h_{3} - h_2}$$

where stations 2 and 3 refer to compressor inlet and outlet, respectively. The compressor adiabatic efficiency measures the minimum work required to achieve a given pressure ratio $\pi_c$ relative to the actual work required to achieve the same pressure ratio. The second efficiency is the polytropic efficiency, $e_c$, defined as

$$e_c = \frac{dh_{3s}}{dh_t}$$

which relates infinitesimal changes in the minimal and actual changes in the stagnation enthalpy. It should be clear that as $\pi_c \to 1$, the two efficiencies become the same. In fact, another name for $e_c$ is the “small stage efficiency”.

From these definitions, please

(a) Draw the isentropic and real compression processes on a Mollier diagram, indicating lines of constant total pressure. Please label the compressor inlet state as ‘2’, the isentropic compressor outlet state ‘3s’ and the nonisentropic compressor outlet state as ‘3’

(b) Using Gibb’s equation applied to the stagnation states,

$$T_i \, ds = dh_i - \frac{dp_i}{\rho_i}$$

show that

$$\frac{dp_i}{\rho_i} = \frac{\gamma e_c}{\gamma - 1} \frac{dT_i}{T_i}.$$  

(c) Integrating the answer from (b) from the compressor inlet to the compressor outlet, show that

$$\pi_c = (\tau_c)^{e_c/(\gamma-1)}.$$  

(d) Using (c) and the definition of $\eta_c$, show that

$$\eta_c = \frac{\pi_c^{(\gamma-1)/\gamma}}{(\tau_c)^{(\gamma-1)/(\gamma e_c)}} - 1.$$  

Problem 5  An axial-flow compressor rotor as an angular velocity of $\omega = 5000$ rpm. The flow entering the compressor rotor has zero preswirl and an axial velocity of 150 m/s. Assuming the axial velocity is constant throughout the stage, and the rotor specific work at the radius 0.5 m is $w_c = 62$ kJ/kg, with $\gamma = 1.4$ and $R = 287$ J/(kg·K), calculate

(a) The stage degree of reaction, $^\circ R$

(b) The total pressure ratio across the rotor at this radius assuming $e_c = 0.9$ and $T_1 = 20^\circ$C.