**Problem 1**

In class we derived the Blasius similarity equation

\[ f''' + ff'' = 0 \]

subject to the boundary conditions \( f(0) = 0, f'(0) = 0, \) and \( f' \to 1 \) as \( \eta \to \infty. \) Our in-class solution used results from symmetry analysis to avoid iterating. Develop one alternative solution method using either (a) a shooting method as we discussed for the Falkner-Skan solution and (b) a matrix-based method described next.

In a matrix method we discretize the domain \( \eta \in [0, \eta_{\text{max}}] \) and use finite difference approximations to estimate the derivatives \( f''' \) and \( f'' \) to the function value \( f. \) For example, if the mesh is uniformly spaced then

\[
 f'_i \approx \frac{f_{i+1} - f_{i-1}}{2\Delta \eta}, \quad f''_i \approx \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta \eta)^2}. 
\]

You will need to derive or find the corresponding expression for \( f'''_i. \) With the derivatives approximated you can write the Blasius ODE as a set of coupled, non-linear algebraic equations

\[
 f'''_i + f_i \left( \frac{f_{i+1} - 2f_i + f_{i-1}}{(\Delta \eta)^2} \right) = 0, 
\]

that must be simultaneously solved by the secant method or some other nonlinear root finder.

**Problem 2**

We showed in class that the temperature equation for incompressible boundary layers can be written

\[
 \rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2. 
\]

Using the Blasius similarity result, show that if the temperature variable is chosen to be

\[
 \Theta = \frac{T - T_e}{T_w - T_e} 
\]

where \( T_e \) is the external flow temperature (outside the BL) and \( T_w \) is the wall temperature, then \( \Theta \) satisfies

\[
 \Theta'' + f(\eta) \Theta' f = -EcPr(f''^2) 
\]

where \( Ec = U_e^2/(C_p(T_w - T_e)) \) and \( f(\eta) \) is the Blasius solution. What are the boundary conditions on \( \Theta \) at \( \eta = 0 \) and as \( \eta \to \infty? \)

**Problem 3**

Numerically solve the temperature similarity solution for the values of \((Ec, Pr) = (0.50, 0.72), (0.25, 0.72), (0, 0.72) \) and \((0, 0.01). \) Comment on what you see.