HW #8
Due Wednesday, April 23, 2014 by 5:00 p.m. to DJB directly

Problem 1  For a steady, quasi-one-dimensional, adiabatic flow without wall friction, what do the following equations simplify to in differential form. For example, the conservation of mass simplifies to \( \frac{d(\rho u A)}{dt} = 0 \).

(a) Conservation of momentum
(b) Conservation of energy
(c) Second law of thermodynamics

Problem 2  A large supply chamber containing air at 6.0 atm and 300 K is connected to a converging nozzle on the left side and a converging-diverging nozzle on the right side. Both nozzles share the same minimum passage area of 100 cm\(^2\). The C-D nozzle has an exit-to-throat area ratio of 1.2.

(a) For the converging nozzle on the left:
   (1) If the ambient pressure \( p_{\text{amb}} \) is reduced to 5.0 atm, what is the mass flow rate in the nozzle?
   (2) How much do we need to lower the ambient pressure to reach the choking point of this converging nozzle?
   (3) What is the corresponding mass flow rate at the choking condition?

(b) Consider the C-D nozzle on the right:
   (1) If the ambient pressure is set at 5.0 atm, do you expect the mass flow rate in the C-D nozzle to be the same as that in the converging nozzle computed before?
   (2) At the C-D nozzle design point, i.e., where the flow is perfectly expanded and supersonic, (i) what is the ambient pressure and (ii) what is the fluid density and velocity at the exit plane?

(c) At the value of \( p_{\text{amb}} \) for which the C-D nozzle is operating on design, calculate the net force exerted on the cart by both nozzles. Towards which direction would the cart begin move?

(d) Suppose that \( p_{\text{amb}} \) varies from 6.0 atm to 0.5 atm. Plot that mass flow rate, \( \dot{m} \), as a function of \( p_{\text{amb}}/p_0 \), for both the converging and C-D nozzles. What do you observe?

Problem 3  Air enters a C-D nozzle which has an exit-to-throat area ratio of 1.8. The back pressure-to-stagnation pressure ratio is \( p_b/p_0 = 0.83088 \). Determine

(a) The location of shock, in terms of \( A_s/A_t \), where \( A_s \) is the area of the nozzle at the shock location
(b) Plot the pressure variation, \( p/p_0 \), with streamwise location
**Problem 4**  The gas turbine engines for fighter aircraft are often equipped with an afterburner to boost their thrust. In the afterburner the hot exhaust gas from the turbine is mixed with new fuel and releases additional heat. This heated gas then flows through the converging-diverging nozzle and exits into the atmosphere at supersonic speed.

The following data are available

- mass flow rate through the engine: \( \dot{m} = 75 \text{ kg/s} \)
- pressure at turbine exit: \( p_1 = 545 \text{ kPa} \)
- temperature at turbine exit: \( T_1 = 830 \text{ K} \)
- Mach number at turbine exit: \( M_1 = 0.3 \).
- back pressure is 101 kPa

Answer the following questions

(a) What is the design Mach number \( M_4 \) of the nozzle when operating without the afterburner?

(b) Estimate the flow area at the turbine exit, \( A_1 \), the throat area \( A_t \), and the nozzle exit area \( A_4 \). Assume the flow is fully isentropic.

(c) If the afterburner is turned on and the stagnation temperature of the exhaust gas is raised to \( T_{03} = 1530 \text{ K} \)

(1) can the same mass flow rate be maintained without changing the throat area?

(2) find the desired throat area with afterburner turned on, and the desired exit area for best thrust.

*Note. You will need to use your Rayleigh flow results.*

![Diagram of flow and area changes](image)

**Problem 5**  Air flows with negligible friction in a constant-area duct. At section 1, properties are \( T_1 = 60 \text{ °C}, \)
\( p_1 = 135 \text{ kPa (abs)}, \) and \( V_1 = 732 \text{ m/s} \). Heat is added between section 1 and section 2, where \( M_2 = 1.2 \). Determine

(a) the properties at station 2

(b) the heat exchange per unit mass

(c) the entropy change