NUMERICAL SIMULATION OF TWO-DIMENSIONAL
ACOUSTIC LINERS WITH HIGH SPEED GRAZING FLOW

BY

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THESIS

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Abstract

The resonant acoustic liner is an effective method to dissipate acoustic energy in engine ducts by converting acoustic fluctuations into vortical fluctuations. To study its characteristics, a fully predictive technique is developed for the flow properties inside and outside the resonator by solving the compressible Navier-Stokes equations with well-posed boundary conditions. Validation of the numerical approach is demonstrated by computing the absorption characteristics of a 3 kHz resonant liner geometry without any background flow. The flow patterns inside and outside the liner are presented. At a fixed sound intensity of 150 dB, the response of the liner to incident acoustic waves with frequencies from 1-6 kHz is investigated in detail. The numerical simulation verifies a previously-held viewpoint that acoustic dissipation is comprised of two frequency-dependent modes: (1) the shedding of small vortices for frequencies below the liner resonant frequency, and (2) the direct viscous dissipation near the liner opening for higher frequencies. The absorption coefficient, which is a measure of the liner’s effectiveness, is evaluated by integrating the viscous dissipation, and is found to be in good agreement with previous numerical and experimental data.

Using the validated approach, simulations with more realistic acoustic liner operating conditions are performed. Incident grazing acoustic waves of different intensity and frequency are performed at different grazing flow Mach number conditions. Compared with previous investigations, the vortex shedding phenomenon is partially different when there is a grazing flow. Studies confirm that incident waves of lower frequency lead to stronger vortex shedding. It is also noted that the vortices shed by the liner apertures are always convected downstream. Moreover, the neck wall shear stress and neck wall displacement thickness have a strong relationship with the vortex shedding. Lastly, the liner’s impedance is evaluated and compared with experimental, theoretical
and empirical data; reasonable agreement is found in most cases.
To father and mother
Acknowledgments

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Nomenclature

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<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
<td>1</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct Numerical Simulation</td>
<td>2</td>
</tr>
<tr>
<td>LES</td>
<td>large-eddy simulation</td>
<td>2</td>
</tr>
<tr>
<td>SDOF</td>
<td>single degree of freedom</td>
<td>2</td>
</tr>
<tr>
<td>2DOF</td>
<td>2 degrees of freedom</td>
<td>2</td>
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<tr>
<td>CAA</td>
<td>Computational Aeroacoustics</td>
<td>3</td>
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<tr>
<td>SPL</td>
<td>Sound Pressure Level</td>
<td>5</td>
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<tr>
<td>DRP</td>
<td>dispersion-relation-preserving</td>
<td>7</td>
</tr>
<tr>
<td>CFL</td>
<td>Courant-Friedrichs-Lewy</td>
<td>20</td>
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<tr>
<td>RK4</td>
<td>fourth order Runge-Kutta</td>
<td>20</td>
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<tr>
<td>NSCBC</td>
<td>Navier-Stokes Characteristic Boundary Condition</td>
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<tr>
<td>NRBC</td>
<td>Non-Reflecting Boundary Condition</td>
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<td>FFT</td>
<td>fast Fourier transform</td>
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List of Symbols

Greek Symbols

\[ \gamma \]  
- Ratio of specific heats  (Page 28)
$\delta_{ij}$  Kronecker delta tensor     (Page 29)
$\delta^*$ Displacement thickness     (Page 46)
$\delta_{99}$ Boundary layer thickness  (Page 42)
$\Delta \phi_i$ Phase correction for position $A_i$  (Page 50)
$\Delta x, \Delta y$ Mesh spacing in $x$ and $y$ direction    (Page 20)
$\epsilon$    Numerical error      (Page 17)
$\zeta$ Planar vorticity      (Page 30)
$\theta$ Normalized acoustic resistance    (Page 48)
$\Theta$ Temperature    (Page 28)
$\kappa$ Thermal conductivity     (Page 28)
$\lambda$ Second coefficient of Newtonian fluid viscosity  (Page 28)
$\lambda$ Wave length     (Page 50)
$\mu$  First coefficient of Newtonian fluid viscosity    (Page 28)
$\rho$ Fluid density      (Page 28)
$\sigma$ Forcing sponge strength parameter     (Page 22)
$\tau_{ij}$ Viscous tensor     (Page 29)
$\phi_{AB}$ Phase difference between signal A and B  (Page 50)
$\chi$ Normalized acoustic reactance  (Page 48)
$\psi$ Planar incident wave angle (made with vertical $+y$ axis)   (Page 29)
$\omega$ Circular frequency, $\omega = 2\pi f$  (Page 29)
$\partial \Omega$ Boundaries of the physical domain     (Page 21)

Roman Symbols

$A$ Amplitude of the plane wave     (Page 29)
$C_p$ Specific heat at constant pressure    (Page 28)
$C_{ph}$ Phase speed     (Page 20)
$C_{abs}$ Absorption coefficient  (Page 33)
$d$ Thickness of the facesheet plate     (Page 47)
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<th>Definition</th>
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<td>$D$</td>
<td>Diameter of the liner aperture</td>
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</tr>
<tr>
<td>$D(x,y)$</td>
<td>Instantaneous viscous dissipation at $(x,y)$</td>
<td>34</td>
</tr>
<tr>
<td>$e$</td>
<td>Resolving efficiency of a finite difference scheme</td>
<td>17</td>
</tr>
<tr>
<td>$E$</td>
<td>Total energy per unit volume</td>
<td>29</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency, $f = \frac{1}{T}$</td>
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<tr>
<td>$H$</td>
<td>Depth of the cavity</td>
<td>49</td>
</tr>
<tr>
<td>$i$</td>
<td>Imaginary unit</td>
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<tr>
<td>$k$</td>
<td>Wavenumber</td>
<td>14</td>
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<tr>
<td>$K$</td>
<td>Kinetic energy carried by each vortex</td>
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<tr>
<td>$\dot{m}$</td>
<td>Mass flow rate</td>
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</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
<td>50</td>
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<tr>
<td>$p$</td>
<td>Pressure</td>
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</tr>
<tr>
<td>$p_i$</td>
<td>Incidence pressure</td>
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</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
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</tr>
<tr>
<td>$q_i$</td>
<td>Heat flux in $ith$ direction</td>
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<tr>
<td>$R$</td>
<td>Acoustic resistance</td>
<td>48</td>
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<tr>
<td>$T$</td>
<td>Period of the oscillation</td>
<td>31</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
<td>28</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>20</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Velocity in the $ith$ coordinate direction</td>
<td>28</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity in the 1st coordinate direction</td>
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<td>$v$</td>
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<tr>
<td>$\mathcal{V}$</td>
<td>Sufficient large integral domain</td>
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<td>$w$</td>
<td>Modified wavenumber</td>
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<td>$w_r$</td>
<td>Real modified wavenumber</td>
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<tr>
<td>$w_i$</td>
<td>Imaginary modified wavenumber</td>
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<td>$X$</td>
<td>Acoustic reactance</td>
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<tr>
<td>$z$</td>
<td>Normalized coustic impedance</td>
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<tr>
<td>$Z$</td>
<td>Acoustic impedance</td>
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Subscripts, Superscripts and Accents

\( \cdot \)\(_\infty \) Environmental or ambient state  (Page 28)

\( \|\cdot\| \) Modulus or Magnitude of the variables  (Page)

\( \cdot^* \) Dimensional variables  (Page 28)

\( \cdot^a \) Acoustic variables  (Page 48)

\( \cdot_L \) Left wall variables  (Page 46)

\( \cdot_R \) Right wall variables  (Page 46)

\( \tilde{\cdot} \) Filtered variables  (Page 23)

\( \overline{\cdot} \) Time-averaged variables  (Page 31)

\( \cdot_{\text{ref}} \) Reference or target state  (Page 22)

\( \cdot' \) First derivative  (Page 13)

\( \cdot'' \) Second derivative  (Page 14)
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4.13 Acoustic impedance prediction of 150 dB incidence. Left: acoustic resistance prediction; Right: acoustic reactance prediction. $- -$ empirical model of Rice and Edward (1971); $- -$ theoretical model of Rice (1976); ○ Present prediction based on point A; ◆ Present prediction based on point A1; ▲ Present prediction based on point A2; △ Present prediction based on point A3; ★ Present prediction based on point A4.
Chapter 1

Introduction

For people living near the vicinity of an airport, the noise from the jet engines can be a nuisance. This problem continues to grow with the increase of air traffic and with the encroachment of residential zones closer to airport boundaries. To reduce aircraft noise pollution, the commercial jet engine manufacturers, National Aeronautics and Space Administration (NASA), and similar European agencies have identified engine noise reduction as one of their near- and long-term goals.

Fig. 1.1 (Batard (2003)) provides a source decomposition of the noise emitted from a typical commercial aircraft. While the total noise is a mixture of engine and airframe sources, the noise from the engine, especially during the take-off process, is considered the main part of the noise pollution. To lower the portion of engine noise created internal to the nacelle, acoustic liners are installed to suppress the acoustic radiation. As one of the main methods to reduce the internal engine noise, acoustic liners are widely installed in the inlets and outlets of the nacelles. Fig. 1.2 (taken from Ref. Batard (2003)) provides a general guideline of the installment of the acoustic liners and Fig. 1.3 zooms in on one of the exhaust ducts. From a noise-reduction perspective, it is beneficial to maximize the number of acoustic liners, but their deployment must be balanced by weight and complexity concerns and by the geometric limitations imposed by high bypass ratio turbofans. Therefore, more effective designs of acoustic liners are required to better serve the goal of noise reduction.

Over the past decades, significant progress in the design and understanding of acoustic liners has been made, mostly through theoretical analysis and experiments. To design better acoustic liners, engineers have examined different liner geometry configurations, testing environments, and construction materials. Through these numerous experiments, many crucial parameters in liner de-
signs have been recognized and applied into manufacturing. In pursuit of an improved understanding of the liner working mechanisms, experiments continue to uncover the fundamental behaviors of liners and their interaction with external flows. Nevertheless, most of these experiments, which are described in more detail in section 1.2.2 below, are expensive and are only just beginning to include the high speed grazing flow environment usually found in practice. Further improvement in non-intrusive flow measurement techniques is on-going.

As computational resources increase high-fidelity simulation techniques, such as direct numerical simulation (DNS) and large-eddy simulation (LES), are becoming prospective candidates for acoustic liner eduction. It is not feasible to simulate the whole engine noise reduction process, but the simulations are useful provided they use reasonable models. A unique advantage of DNS and LES is the availability of non-intrusive space-time data.

The aim of the work documented herein is to present a predicative approach in acoustic liner performance in the presence of high-speed grazing flow through DNS. Validation cases are performed to check the fidelity of the method before it is applied to the more realistic simulation environment.

1.1 Background

Conventional aircraft engine duct acoustic liners can be classified into three types (see Fig. 1.4 (Motsinger and Kraft (1991))): (1) single degree of freedom (SDOF) (2) 2 degrees of freedom (2DOF) (3) bulk absorber. The SDOF liner is a perforated face sheet with honeycomb resonators and solid backplate on the bottom of the cavities. The 2DOF liner adds a second layer with septum sheet (mid sheet). Bulk absorbers, in contrast, are characterized by a singler layer with a porous face sheet on the top and a backplate on the bottom, with porous materials (e.g., fibrous mat file) in between the face sheet and backplate. According to Motsinger and Kraft (1991), SDOF liners are most effective for narrow frequency ranges and bulk absorbers are more effective for broadband sound. However, due to construction difficulties, bulk absorber liners are seldom applied in commercial aircraft.

As there exists a wide range of possible factors that may affect the performance of acoustic
liners, most studies have been restricted to the SDOF liners. Relatively simple SDOF liners exhibit complicated fluid mechanical behavior that is not completely understood. A survey of the investigative methods used for liners is given below:

**Theoretical Approach**  The theoretical design procedures represent ideal approaches for the analysis of duct acoustic propagation and radiation. However, these methods require knowledge of the source modal characteristics, which are difficult to estimate without careful experiments. When there is little information about the source characteristics, either because the particular turbomachinery is still in the early design stage or component test data are not available, theoretical approaches are often only rough approximations.

**Empirical Approach**  The empirical approach is established on extensive experimental testing. They are the most reliable way to provide data for particular designs. However, extensive testing can be time consuming and expensive and the tests are not always representative of the real installation environment. Extrapolation of empirical data to novel designs is challenging.

**Semi-empirical Approach**  In the semi-empirical approach, the theoretical and empirical procedures are combined to provide a rational prediction based on experience when source modal characteristics are unknown, at the expense of rigor. The success of semi-empirical methods rests on the modeling assumptions made and on the available experimental data. Most liners currently under consideration were designed by a semi-empirical process.

**Numerical Approach**  With the increase of computational power, Computational Aeroacoustics (CAA) is emerging as a tool capable of predicting liner performance. Here, the compressible Navier-Stokes equations are solved directly and the results are post-processed similar to the experimental data. The input parameters (eg., liner geometries) can be easily changed, and the simulations may include realistic conditions. However, one disadvantage of numerical liner eduction method is the long wall-clock time required for each computation. Increasingly powerful computers reduce the cost but careful selection must still be exercised to maximize available resources. Further, highly accurate simulation techniques with complex geometries capabilities are still an active area of research.
1.2 Literature Review

A brief survey of the existing literature follows.

1.2.1 Theoretical Prediction of Acoustic Liner Characteristics

Hersh and Rogers (1976) developed a fluid mechanical model of the acoustic behavior of small orifices. The model was able to predict orifice resistance and reactance as a function of incident sound pressure level, frequency, and orifice geometry. Several important conclusions were:

1. Locally spherical acoustic flow was found in the immediate neighborhood of the orifice.

2. Viscous effects dominate the orifice resistance and the orifice reactance is directly related to the inertia of the oscillating flow in the neighborhood of the orifice.

3. The acoustic impedance is dominated by nonlinear jet-like effects.

Rice (1976) presented an analysis of the oscillatory fluid flow in the vicinity of a circular orifice similar to Hersh and Rogers (1976) but with an additional steady grazing flow. It was noted that the equations are approximately linear in the region where the grazing flow effects are dominant and therefore a solution of the acoustic impedance for this region was presented.

Howe (1979) developed an expression for the Rayleigh conductivity of an aperture through which a high-Reynolds-number flow passes. Viscous effects were limited to the separation of the flow from the rim of the aperture. The harmonic pressure difference across the orifice thus leads to the periodic shedding of vorticity, and consequently acoustic energy as converted into mechanical energy, which is subsequently dissipated into heat. Howe modeled the energy dissipation mechanism as linear, in the sense that the fraction of incident sound energy that is absorbed is independent of the amplitude of the sound.

1.2.2 Experimental Observations of Acoustic Liner Performance

Ingard and Labate (1950) found that when the opening size of the acoustic liner was large, the viscous effects could be significantly different from liners with small apertures. Moreover, for liners
with small apertures, they noted that the oscillatory turbulent jet model might not be applicable.

The original idea to calculate the impedance of a liner by applying a semi-empirical method can be traced back to Melling (1973), who realized that the linear or nonlinear behavior of the excited flow in the liner depends mostly on the sound pressure level (SPL). Melling proposed that for low intensity sound waves, wall friction around the resonator opening is the principle dissipation mechanism, while for high intensity sound waves an oscillatory turbulent jet was found right at the entrance of the resonator.

Dean (1974) first proposed the two-microphone method for measuring the acoustic impedance of the duct liners which has found wide application because of its simplicity and reliability. This technique is shown to be capable of impedance evaluations over a range of flow conditions to an accuracy of 10% or better and can be applied when a grazing flow exists. The measurement limitations were thoroughly investigated and the relative importance of the sources of errors in the measurement technique has been quantified.

Zorumski and Tester (1979) reviewed the prediction of the acoustic impedance of duct liners, which includes both linear and nonlinear and the effect of grazing flow properties of sheet and bulk type materials and method for the measurement of these properties. Several methods for predicting the properties of single or multilayered, point reacting or extended reaction and flat or curved liners were also discussed.

Hughes and Dowling (1990) and Dowling and Hughes (1992) studied the vortex shedding mechanism of screens with regular arrays of slits and circular perforations with mean bias flow, respectively. They showed that it is theoretically possible to absorb all impinging sound at a particular frequency if a rigid backing wall is included, such that reflection from the wall allows substantially more interaction between the sound and screen.

Jing and Sun (1999) investigated experimentally perforated liners with bias flow and found that the presence of the bias flow can markedly increase both the absorption coefficient and effective bandwidth of a perforated liner. They also showed that the plate thickness has a major influence on the acoustic properties of a liner with bias flow. A simple empirical model was proposed with good agreement of the experimental data.
Tam et al. (2001) performed experiments to validate the numerical simulations of Tam and Kurbatskii (2000), in which an acoustic driver was placed 21.625 inches from orifice to produce high intensity sinusoidal incident acoustic waves. Two microphones were flush mounted near the reference plane where the orifice was placed to determine acoustic impedance. Particle image velocimetry was used across the width of the slit-orifice and cavity to visualize the possible vortex formation. The absorption coefficient (fraction of total sound power absorbed), given by

\[ C_{abs} = \frac{E_{dissipation}}{E_{incident}} \cdot \frac{A_s}{A_t} \]  

was measured. Good agreement between the numerical and experimental absorption coefficients were found in most cases and confirmed the previously-held view that vortex shedding mechanism is far more efficient in acoustic noise suppression than pure viscous dissipation.

Malmary, Carbonne, Aurégan, and Pagneux (2001) presented some of the existing models for perforated plates impedance and compared them with the empirical models. To test the acoustic impedance of the perforated plates, both the traditional two microphone method and the moving microphone method were presented. Comparisons revealed that while the traditional two microphone method is easier to use and has a better agreement with the previous models, the moving microphone method is able to provide more pressure field information across the duct.

Jing, Sun, Wu, and Meng (2001) conducted a set of studies with grazing flow over perforated plates with different orifice geometries and plate thicknesses. The intensities of the acoustic incident waves were kept low enough to avoid nonlinear effects. The two microphone method was adopted to test the acoustic impedance of the perforated plates. Good agreement was found between the experimental data and their semi-empirical formula.

Eldredge and Dowling (2003) conducted the experiment concerning the effectiveness of a cylindrical perforated liner with mean bias flow in its absorption. A one-dimensional model of the absorption mechanism was developed and a homogeneous liner compliance was utilized. The model was evaluated by comparing with experimental results and excellent agreement was found. It is noted that their system can absorb a large fraction of incoming energy, and can prevent all of the
energy produced by an upstream source in certain frequency ranges from reflecting back of planar acoustic waves. Moreover, the increase of the bandwidth of this strong absorption can be achieved by appropriate placement of the liner system in the duct.

1.2.3 Numerical Predictions of Acoustic Liner Characteristics

Tam and Kurbatskii (2000) performed a series of numerical simulations of the fluid dynamics around a resonant liner aperture. The primary objective of the investigation was to give a better understanding of the flow fields and physics around the openings of liner resonators when they are excited by incident acoustic waves at different sound intensities. A seven-point dispersion-relation-preserving (DRP) scheme (Tam and Webb (1993)) was applied to minimize numerical dispersion and dissipation. Results revealed two different acoustic energy convert mechanisms of different incident waves. Vortex shedding is found in most of the high intensity incident waves (SPL $\geq$ 160dB) and at moderate intensity incident waves (SPL $\in$ (140,150)dB) at low frequencies less than the frequency of the resonator. It was also noted that vortex shedding occurs near but not necessarily at the resonance frequency and high SPL is required for highly nonlinear flow phenomenon.

Tam, Ju, Jones, and Parrott (2005) performed a numerical and experimental study of slit liners with different geometries (slit angles), slit widths and discrete sound frequencies, in which extensive comparisons between experimental measurements and DNS results of the interaction of acoustic waves and a slit resonator are provided. They observed that two large vortices are shed from 45° beveled slits regardless of whether the fluid is entering or exiting the cavity. These two large vortices are always followed by two thin shear layers, which are inviscidly unstable. The instability grows very quickly and causes the thin layer to roll up into a string of tiny vortices. Thus, for 90° corners, vortex shedding ceases when SPL is reduced to some level but for 45° corners, vortex shedding will continue until some lower SPL is achieved. It was noted that resonators with 45° corners are more efficient in acoustic energy dissipation than 90° corners since more vortices are generated and shed by the boundaries of the aperture.

Leung, So, Wang, and Li (2007) performed an investigation of sound absorption by an in-duct orifice with a sixth-order finite difference Padé scheme with explicit fourth-order time marching
Runge-Kutta method to solve the governing Navier-Stokes equations. They concluded that the small orifices are more effective in sound absorption whereas a larger opening is nearly transparent to the incident waves.

Eldredge, Bodony, and Shoeybi (2007) performed a numerical simulation of a turbulent flow through an aperture in a multi-perforated liner with incompressible LES. By comparing with the theoretical model of Howe (1979), the simulation results of Rayleigh conductivity of the aperture provides a reasonable estimate of the LES-calculated impedance at small frequencies but loses accuracy at higher frequencies.

The introduction of laminar boundary layer as a base flow in 2-D slit resonator numerical simulation was performed by Tam, Ju, and Walker (2008) with corresponding validation experiments. The flow field outside the cavity given by Tam’s numerical simulation had a good agreement with Walker’s flow visualization experiments data. Moreover, simulation results showed that vortices may merge together and convect downstream by the grazing flow for a long distance, which are likely to influence the flow of neighboring resonators.

Tam, Ju, Jones, Watson, and Parrott (2009) performed a computational and experimental investigation of the acoustic properties of a three-dimensional slit resonator. Their findings show that the shed vortices behavior in three dimensions is quite different from those of two dimensions. It is observed that the shed vortices tend to evolve into ring vortices and align themselves into two regularly spaced vortex trains moving away from the resonator opening in opposite directions. The phenomenon is different from the two dimensional simulations in which chaotic shedding behaviors are found. Despite the different vorticity dynamics, investigations again confirms that the acoustic energy dissipation is dominated by vortex shedding mechanism for high incident SPL.

### 1.3 The Structure of the Thesis

The essential numerical schemes performed in this work with appropriate initial and boundary condition are discussed in Chapter 2. The following two chapters then present the detailed simulation cases. In Chapter 3, the simulation focuses on normal incident waves with different frequencies in
a quiescent medium. Based on the validation results in Chapter 3, a set of more realistic cases are discussed in Chapter 4, in which a grazing flow with grazing incidence is added (see Fig. 1.5). Conclusions and proposed future work are presented in Chapter 5.

1.4 Figures for Chapter 1

Figure 1.1: Distribution of the aircraft noise sources in the process of taking-off and landing (Batard (2003)).
Figure 1.2: Schematic of the installment of acoustic liners in a jet engine (Batard (2003)).

Figure 1.3: Schematic of acoustic liners in one of the exhaust walls in an engine.
Figure 1.4: Schematic of three acoustic liner designs (Motsinger and Kraft (1991)).
Figure 1.5: Schematic figures of no flow normal incidence (a), grazing flow normal incidence (b) and grazing flow grazing incidence (c).
Chapter 2

Numerical Methods of High Order Finite Difference Schemes

A wide range of space and time scales are common in many physical phenomena; acoustic waves propagating in a compressible flow is a typical example. To apply DNS to these phenomena, all the relevant scales should be taken into consideration for well-resolved solutions. Different numerical schemes have different resolution characteristics. The compact finite difference schemes (Padé schemes) presented in the following sections behave differently (especially in the shorter scales) compared to more traditional explicit finite difference approximations. In the field of computational aeroacoustics where low dissipative and dispersive schemes are required, these compact finite difference schemes also have more resolving power compared to the traditional explicit lower order schemes.

2.1 Formulation of Compact Finite Difference Schemes

2.1.1 Approximation of the Derivatives

For simplicity, only a uniformly spaced mesh is considered, where $h$ is the mesh spacing and $x_i = h(i - 1), 1 \leq i \leq N$, is the $i^{th}$ node location. The function value $f_i(x) = f(x_i)$ is simplified by $f_i$ and $f'_i$, $f''_i$ represents the first and second derivative at the node $i$, respectively. The finite difference approximation to the derivative of the function can be expressed as a linear combination of neighboring function values:

$$f'_m \approx \sum_{i=-p}^{q} a_i f_{m+i}$$

(2.1)
The matrix form of Eqn. 2.1 can be written as,

\[ P \mathbf{f}' = Q \mathbf{f} \quad (2.2) \]

where the coefficient matrices \( P \) and \( Q \) determine the finite difference scheme. It is always possible to increase the order of scheme accuracy at the expense of making denser coefficient matrices \( P \) and \( Q \). Note that if \( P = I \), the schemes are explicit.

The expression

\[ \beta f'_{i-2} + \alpha f'_{i-1} + f'_{i} + \alpha f'_{i+1} + \beta f'_{i+2} = \frac{c}{6h} f_{i+3} - \frac{f_{i-3}}{6h} + \frac{b}{4h} f_{i+2} - \frac{f_{i-2}}{4h} + \frac{a}{2h} f_{i+1} - \frac{f_{i-1}}{2h} \quad (2.3) \]

generalizes the first order derivative up to tenth order accuracy (Lele (1992)). The relations between the coefficient \( a, b, c \) and \( \alpha, \beta \) can be derived by matching the Taylor series coefficients of various orders. Similarly, the derivation of the compact approximation for the second derivative (again up to tenth order accuracy) can be written as,

\[ \beta f''_{i-2} + \alpha f''_{i-1} + f''_{i} + \alpha f''_{i+1} + \beta f''_{i+2} = \frac{c}{9h^2} f_{i+3} - \frac{2f_{i+1} + f_{i-3}}{9h^2} + \frac{b}{4h^2} f_{i+2} - \frac{2f_{i+1} + f_{i-2}}{4h^2} + \frac{a}{h^2} f_{i+1} - \frac{2f_{i+1} + f_{i-1}}{h^2} \quad (2.4) \]

The coefficients of the first and second derivatives for a variety of schemes are tabulated in Table 2.1 and Table 2.2.

**2.1.2 Fourier Analysis of Finite Difference Operators**

We assume the variables to be periodic over the domain \([0, L]\) of the independent variable \(x\):

\[ f(x) = \sum_{k=-N/2}^{k=N/2} \hat{f}_k e^{\frac{2\pi ikx}{L}} \quad (2.5) \]

where \(i = \sqrt{-1}\).

Define a scaled wavenumber by \(w = 2\pi kh/L\) and a scaled coordinate \(s = x/h\). The Fourier
Table 2.1: Coefficients for the first derivative schemes of finite difference operators.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b)</td>
<td>0</td>
<td>0</td>
<td>$\frac{4}{3}$</td>
<td>$-\frac{1}{3}$</td>
<td>0</td>
</tr>
<tr>
<td>(c)</td>
<td>0</td>
<td>0</td>
<td>$-1.5$</td>
<td>0.6</td>
<td>$-0.1$</td>
</tr>
<tr>
<td>(d)</td>
<td>0.25</td>
<td>0</td>
<td>1.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(e)</td>
<td>$\frac{1}{3}$</td>
<td>0</td>
<td>$\frac{14}{9}$</td>
<td>$\frac{1}{9}$</td>
<td>0</td>
</tr>
<tr>
<td>(f)</td>
<td>$\frac{3}{8}$</td>
<td>0</td>
<td>$\frac{25}{16}$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{1}{80}$</td>
</tr>
<tr>
<td>(g)</td>
<td>$\frac{5}{12}$</td>
<td>0</td>
<td>$\frac{29}{18}$</td>
<td>$\frac{2}{9}$</td>
<td>0</td>
</tr>
<tr>
<td>(h)</td>
<td>0.5</td>
<td>$\frac{1}{20}$</td>
<td>$\frac{17}{12}$</td>
<td>$\frac{101}{150}$</td>
<td>$\frac{1}{100}$</td>
</tr>
<tr>
<td>(i)</td>
<td>0.5771439</td>
<td>0.0896406</td>
<td>1.3025166</td>
<td>0.9935500</td>
<td>0.03750245</td>
</tr>
</tbody>
</table>

modes in terms of $s$ and $w$ are simply $e^{iw}$ times. The domain of the scaled wavenumber $w$ is $[0, \pi]$. The exact first derivative generates a function with Fourier coefficients $\hat{f}'_k = iw\hat{f}_k$. The differencing error of the first derivative scheme may be assessed by comparing the Fourier coefficients of the derivative obtained from the differencing scheme $\hat{f}'_k$ with the exact Fourier coefficients $\hat{f}'_k$. Exact differentiation corresponds to the straight line $w' = w$.

The modified wavenumber of the first order derivative difference scheme (Eqn. 2.3) is given by the following formula (Lele (1992))

$$w'(w) = \frac{a \sin(w) + b/2 \sin(2w) + c/3 \sin(3w)}{1 + 2\alpha \cos(w) + 2\beta \cos(2w)}$$ (2.6)

The plot of Eqn. 2.6 as a function of the wavenumber is presented in Fig. 2.1 for a variety of schemes, whose coefficients are given in Table 2.1. These schemes are mostly used in real
Table 2.2: Coefficients for the second derivative schemes of finite difference operators.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b)</td>
<td>0</td>
<td>0</td>
<td>$\frac{4}{3}$</td>
<td>$-\frac{1}{3}$</td>
<td>0</td>
</tr>
<tr>
<td>(c)</td>
<td>0</td>
<td>0</td>
<td>1.5</td>
<td>$-0.6$</td>
<td>0.1</td>
</tr>
<tr>
<td>(d)</td>
<td>0.1</td>
<td>0</td>
<td>1.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(e)</td>
<td>$\frac{2}{11}$</td>
<td>0</td>
<td>$\frac{12}{11}$</td>
<td>$\frac{3}{11}$</td>
<td>0</td>
</tr>
<tr>
<td>(f)</td>
<td>$\frac{9}{38}$</td>
<td>0</td>
<td>$\frac{147}{152}$</td>
<td>$\frac{51}{95}$</td>
<td>$-\frac{23}{760}$</td>
</tr>
<tr>
<td>(g)</td>
<td>$\frac{344}{1179}$</td>
<td>$\frac{23}{2358}$</td>
<td>$\frac{320}{393}$</td>
<td>$\frac{310}{393}$</td>
<td>0</td>
</tr>
<tr>
<td>(h)</td>
<td>$\frac{334}{899}$</td>
<td>$\frac{43}{1798}$</td>
<td>$\frac{1065}{1798}$</td>
<td>$\frac{1038}{899}$</td>
<td>$\frac{79}{1798}$</td>
</tr>
<tr>
<td>(i)</td>
<td>0.50209266</td>
<td>0.05569169</td>
<td>0.21564935</td>
<td>1.72332200</td>
<td>0.17659730</td>
</tr>
</tbody>
</table>

applications. It is clear that all the schemes are better solved for longer waves than shorter waves and they all fail at $w = \pi$.

The error analysis for the second and higher derivative approximations are similar to the analysis for the first derivative. The exact second derivative of the fourier analysis formula generates a function with Fourier coefficients $\hat{f}''_k = -w^2 \hat{f}_k$. The numerical approximations correspond to $(\hat{f}''_k)_{fd} = -w'' \hat{f}_k$ is given by

$$w''(w) = \frac{2a(1-\cos(w)) + b/2(1-\cos(2w)) + 2c/9(1-\cos(3w))}{1 + 2a\cos(w) + 2\beta\cos(2w)}$$  \hspace{1cm} (2.7)

Fig. 2.2 shows the modified wave number plotted against the wavenumber of the exact second derivative Fourier analysis. It is clear that one can reach similar conclusions from the first derivative approximation Fourier analysis.
It may also noted that different schemes deviate from the exact solution at different wavenumbers and to quantify this we need the following definition. For the quantitative analysis, we define the error tolerance as:

\[ \frac{|w'(w) - w|}{w} \leq \epsilon \quad \text{1st derivative} \quad \frac{|w''(w) - w^2|}{w^2} \leq \epsilon \quad \text{2nd derivative} \]

The shortest well resolved wave \( w_f \), certainly depends on the tolerance error, and separates the well resolved and poorly resolved waves. Therefore, the fraction \( e_i \approx \frac{w_f}{\pi} = 1 - r_i \) can be regarded as a measure of the resolving efficiency of a finite difference scheme. The resolving efficiency obviously depends not only of the scheme itself but also has a strong relation with the error tolerance.

Table 2.3 and Table 2.4 list the solving efficiency under three different values of the error tolerance \( \epsilon \) for the first and second derivatives finite difference schemes approximations. The tabulated results suggest that the trend of increasing resolving efficiency as the schemes increase the accuracy order. It is evident that even if two schemes has the same accuracy order, the different parameters gives different solving efficiencies. It may also be noted that the “spectral-like” scheme (Table 2.1 (i)), which is obtained by imposing fewer order-of-accuracy constraints and additional wavenumber constraints, has a dramatic improvement in resolution characteristics.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>( \epsilon = 0.1 )</th>
<th>( \epsilon = 0.01 )</th>
<th>( \epsilon = 0.001 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.25</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>(b)</td>
<td>0.44</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>(c)</td>
<td>0.54</td>
<td>0.35</td>
<td>0.23</td>
</tr>
<tr>
<td>(d)</td>
<td>0.59</td>
<td>0.35</td>
<td>0.20</td>
</tr>
<tr>
<td>(e)</td>
<td>0.70</td>
<td>0.50</td>
<td>0.35</td>
</tr>
<tr>
<td>(f)</td>
<td>0.86</td>
<td>0.58</td>
<td>0.45</td>
</tr>
<tr>
<td>(g)</td>
<td>0.77</td>
<td>0.60</td>
<td>0.32</td>
</tr>
<tr>
<td>(h)</td>
<td>0.81</td>
<td>0.68</td>
<td>0.56</td>
</tr>
<tr>
<td>(i)</td>
<td>0.90</td>
<td>0.84</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 2.3: Resolving Efficiency \( e_i(\epsilon) \) of the First Derivative Schemes.

For the boundary schemes, we derive the first derivative at the boundary \( i = 1 \) in the following
### Table 2.4: Resolving Efficiency $\epsilon_1(\epsilon)$ of the Second Derivative Schemes.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$\epsilon = 0.1$</th>
<th>$\epsilon = 0.01$</th>
<th>$\epsilon = 0.001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.35</td>
<td>0.11</td>
<td>0.03</td>
</tr>
<tr>
<td>(b)</td>
<td>0.60</td>
<td>0.32</td>
<td>0.17</td>
</tr>
<tr>
<td>(c)</td>
<td>0.71</td>
<td>0.45</td>
<td>0.29</td>
</tr>
<tr>
<td>(d)</td>
<td>0.69</td>
<td>0.39</td>
<td>0.22</td>
</tr>
<tr>
<td>(e)</td>
<td>0.81</td>
<td>0.56</td>
<td>0.38</td>
</tr>
<tr>
<td>(f)</td>
<td>0.87</td>
<td>0.65</td>
<td>0.49</td>
</tr>
<tr>
<td>(g)</td>
<td>0.89</td>
<td>0.67</td>
<td>0.51</td>
</tr>
<tr>
<td>(h)</td>
<td>0.92</td>
<td>0.74</td>
<td>0.60</td>
</tr>
<tr>
<td>(i)</td>
<td>1.00</td>
<td>0.90</td>
<td>0.85</td>
</tr>
</tbody>
</table>

With this choice the boundary schemes can be used with a tridiagonal interior scheme without increasing the bandwidth.

The coefficients of the first order boundary derivatives for a variety of the schemes are tabulated in Table 2.5. As discussed in the Fourier analysis of the interior finite difference schemes, the boundary derivatives can also be analyzed in a similar way. However, it should be noted that the Fourier analysis of the boundary approximations can be justified only at a heuristic level, while the application of the analysis to the interior differencing with periodic boundary conditions is more rigorous.

The modified wavenumber $w'$ in boundary derivatives Fourier analysis is generally complex and it is reasonable to split it into the real part indicated by $w'_r$ and the imaginary part indicated by $w'_i$. While the real part of the modified wavenumber is always associated with the dispersive error, the imaginary part is more related to the dissipative error. Fig. 2.3 and Fig. 2.4 show the real and imaginary parts of the modified wavenumber against the wavenumber for the various boundary finite difference schemes for the first order derivatives. It should be noted that the increasing of the formal accuracy of the explicit approximations reduces the dissipative error in the low-intermediate wavenumber range, but at the same time time degrades the the dispersive error for the intermediate...
Table 2.5: Coefficients for the boundary derivative schemes of finite difference operators.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$\alpha_b$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b)</td>
<td>0</td>
<td>$-\frac{3}{2}$</td>
<td>2</td>
<td>$-\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>(c)</td>
<td>0</td>
<td>$-\frac{11}{6}$</td>
<td>3</td>
<td>$-\frac{3}{2}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>(d)</td>
<td>1</td>
<td>-2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(e)</td>
<td>2</td>
<td>$-\frac{5}{2}$</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>(f)</td>
<td>3</td>
<td>$-\frac{17}{6}$</td>
<td>3</td>
<td>$\frac{3}{2}$</td>
<td>$-\frac{1}{6}$</td>
</tr>
<tr>
<td>(g)</td>
<td>5</td>
<td>$-\frac{7}{2}$</td>
<td>1</td>
<td>$-\frac{7}{2}$</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>(h)</td>
<td>4</td>
<td>$-\frac{3}{2}$</td>
<td>3</td>
<td>$-\frac{1}{2}$</td>
<td></td>
</tr>
</tbody>
</table>

wavenumbers. The second-order compact scheme with $\alpha = 1$ and $d = 0$ is purely non-dissipative but it has a singular $w'_r$ at $w = \pi$. The third-order compact scheme with $\alpha = 0$ and $d = 0$ has very small dispersive error and its dissipative error (shown by $w'_r$) is also confined to high wavenumbers. It should be noted that the third-order compact scheme ($\alpha = 5$) and second-order compact scheme ($\alpha = 5, d = 0.5$) share a very low-dissipative character, but their dispersive (shown by $w_r$) do not have a better resolution than the third-order compact scheme ($\alpha = 2, d = 0$).

It should also be noted that for many of these compact boundary schemes described above their $w'_r$ may have a sign opposite to that obtained with the explicit one-sided boundary formulas. Note in Fig. 2.4 that the value $w'_r(\pi)$ of the first, second and third-order explicit ($\alpha = 0$) boundary schemes are 2, 4 and $\frac{20}{3}$ respectively, while the third and fourth-order compacts schemes (with $\alpha = 2$ and $\alpha = 3$, respectively) have negative values of $w'_r(\pi)$.
2.1.3 Phase Speed

The dispersive error characteristics can be presented in an alternative way which focuses in terms of the error in the phase speed of waves of different wavenumber. Consider the simple scalar transport equation

\[ \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} = 0 \]

in which the time advancement is treated exactly. The phase speed for a wave is given by the finite difference scheme as \((c_p)_{fd} = \frac{w'}{w}\). The exact solution has unit phase speed and \((c_p)_{fd} - 1\) is thus the measure of the phase error.

Fig. 2.5 presents the phase speed of the different finite difference schemes. It is quite similar to the modified wave plots except that all the finite difference schemes “converge to” zero phase speed at \(w = \pi\) while the exact phase speed is 1. It is also clear lower order schemes may deviate from the exact phase speed at a relatively low wavenumber while higher order schemes may increase \(C_{ph}\) above 1.

2.1.4 Application

Based on the previous analysis and considering feasibility of the computational wall clock time, the simulations in Chapter 3 and Chapter 4 apply the sixth order accuracy Padé scheme (scheme (e) in Table 2.1) with fourth order accuracy boundary scheme (scheme (f) in Table 2.5).

2.2 Time Marching Schemes

The time integration uses the standard non-linear fourth order of Runge-Kutta (RK4) scheme either running at a constant Courant-Friedrichs-Lewy (CFL) condition or a constant time step. The definition of the CFL condition in 2-D is given by

\[
\text{CFL} = \Delta t \left\{ \frac{|u|}{\Delta x} + \frac{|v|}{\Delta y} + c \sqrt{ \frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} } \right\}
\] (2.9)
where $\Delta t$ is the time step, $u$ and $v$ correspond to the velocity in $x$ and $y$ direction, $\Delta x$ and $\Delta y$ are the mesh spacing in $x$ and $y$ direction, respectively.

The details of the RK4 scheme are given by

\[
\begin{align*}
    k_1 &= h_t f(x_n, y_n) \\
    k_2 &= h_t f(x_n + \frac{1}{2} h, y_n + \frac{1}{2} k_1) \\
    k_3 &= h_t f(x_n + \frac{1}{2} h, y_n + \frac{1}{2} k_2) \\
    k_4 &= h_t f(x_n + h, y_n + k_3) \\
    y_{n+1} &= y_n + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) + O(h_t^5),
\end{align*}
\]

(2.10)

where $h_t$ is the time step. The condition $\text{CFL} \leq 0.8$ is always satisfied to keep the stability of these simulations.

### 2.3 Boundary Conditions

In the numerical simulations, the following 3 types of boundary conditions are always applied:

- **Isothermal no-slip boundary:** These boundary conditions work for many engineering flows and can be written as

\[
\begin{align*}
    \mathbf{u}(x, t) &= \mathbf{u}_{\text{wall}}(t) \quad x \in \partial \Omega \\
    \Theta(x, t) &= \Theta_{\text{wall}}(t) \quad x \in \partial \Omega
\end{align*}
\]

(2.11) (2.12)

where $\mathbf{u}_{\text{wall}}(t)$ and $\Theta_{\text{wall}}(t)$ are the specified wall velocity and temperature, respectively, on the boundary $\partial \Omega$.

- **Non-reflection boundary:** DNS of compressible flows requires an accurate control of wave reflections from the boundaries of the computational domain. As these unwanted waves should be eliminated as much as possible without adding too much numerical dissipation, a
better-nonreflecting treatment of the boundary is required. Thompson (1987) extends the concept of non-reflecting boundary conditions (NRBC) to the multidimensional case in non-rectangular coordinate systems and results of several typical fluid dynamics problems were provided. Studies NRBC by Rowley and Colonius (2000) gives higher-order accuracy for linear hyperbolic systems in multi-dimensional domain. These discrete NRBCs ensure well-posedness condition and the spurious reflection at boundary can be quantified. Poinsot and Lele (1992) investigated the Navier-Stokes Characteristic Boundary Condition (NSCBC) for both Euler and Navier-Stokes equations. The NSCBC strategies for Euler Equations was described in detail and with supplement of additional viscous conditions, the NSCBC Strategy for the Navier-Stokes equations was configured.

- **Sponge**: The application of sponge regions, which add the forcing term \(-\sigma(x)(q - q_{\text{ref}})\) to the right-hand-side of the governing equations, aims at forcing a solution towards a target state \(q_{\text{ref}}(x, t)\). Based on the functions of the sponge zones, they can be classified into absorbing sponges and forcing sponges. The absorbing sponges are used to absorb and minimize reflections from computational boundaries, which are helpful to the stability of the numerical simulations. The forcing sponges, however, try to introduce prescribed disturbances, either steady or unsteady, into a calculation. Note \(\sigma(x)\) is a function of location \(x\). The form of \(\sigma(x)\) in the simulations is chosen to be

\[
\sigma(x) = \sigma_{\text{max}}|x - x'|^2
\]

(2.13)

where \(\sigma_{\text{max}}\) is a parameter, \(x'\) denotes the terminal location of the sponge zone. Detailed analysis of the sponge zone analysis for computational fluid dynamics can be found in Bodony (2006).

An application of these boundary conditions are presented in Fig. 2.6. The thick solid lines denote isothermal boundary conditions and the dash lines are the non-reflecting boundaries. The shadow region represents the sponges zones: the left sponge plays a role of forcing inflow conditions or perturbations while the right one aims at damping unwanted waves from that may reflect back
from the boundaries.

2.4 Numerical Filtering

To make the simulation feasible, non-uniform meshing is always required to control the number of meshing points. According to Visbal and Gaitonde (2002), the application of the high-order compact scheme to nonsmooth meshes without filtering incorporation will result in spurious oscillations which inhibit their applicability. One way to remove the non-physical oscillations is the introduction of the discriminating low-pass high-order filter, which restores the advantages of high-order approach even in the presence of large grid discontinuities. If a component of the solution vector is denoted by \( f \), filtered values \( \hat{f} \) are obtained by solving the tridiagonal system

\[
\alpha_f \hat{f}_{i-1} + \hat{f}_i + \alpha_f \hat{f}_{i-1} = \sum_{n=0}^{N} \frac{a_i}{2} (f_{i+n} + f_{i-n}) \tag{2.14}
\]

with proper choice of the coefficients \( a_i \). Eqn. 2.14 provides a \( 2N \)th-order filtering. Note in Eqn. 2.14 \( \alpha_f \) in is a free coefficient between \([-0.5, 0.5]\) for numerical stability. Coefficients \( a_i \), \( (i = 0, 1, \cdots, N) \) are derived in terms of \( \alpha_f \) and different expressions for \( a_i \) were found for different stencils. Note that the order of the numerical filtering should be at least two orders higher than the finite difference schemes applied in the simulation. The simulations performed in Chapter 3 and Chapter 4 applies a tenth order filtering (four orders higher than the sixth order Padé scheme) with the choice of \( \alpha_f = 0.49 \) and \( \alpha_f = 0.2 \), respectively. Detailed expressions of \( a_i \) can be found in Visbal and Gaitonde (2002).

2.5 Figures for Chapter 2
Figure 2.1: Reproduction of Lele (1992) plot of modified wavenumber for first derivative approximation.

Figure 2.2: Reproduction of Lele (1992) plot of modified wavenumber for second derivative approximation.
Figure 2.3: Reproduction of Lele (1992) plot of real part of modified wavenumber for the first derivative boundary schemes.

Figure 2.4: Reproduction of Lele (1992) plot of imaginary part of modified wavenumber for the first derivative boundary schemes.
Figure 2.5: Reproduction of Lele (1992) plot of phase speed for the first derivative approximations.

Figure 2.6: Schematic of the application of three types of boundary conditions. Legend: —, isothermal no-slip boundary; ——, nonreflecting boundary; shadow region, sponge zones.
Chapter 3

Validation for a 2-D Resonant Liner with Normal Incidence

3.1 Physical Model

3.1.1 Introduction

Fig. 3.1 shows a liner exposed to a normally incident acoustic wave in a quiescent medium. The aim of this study is to develop a validated, fully predictive technique for computing the absorption characteristics by solving the compressible Navier-Stokes equations in a 3 kHz resonant liner geometry. The flow patterns inside and outside the liner are presented. At a fixed sound intensity of 150 dB, the response of the liner to incident acoustic waves with frequencies from 1-6 kHz is investigated in detail. The numerical simulation verifies a previously-held viewpoint that acoustic dissipation is comprised of two frequency-dependent modes: (1) the shedding of small vortices for frequencies below the liner resonant frequency, and (2) the direct viscous dissipation near the liner opening for higher frequencies. For each of these cases the absorption coefficient is computed by integrating the viscous dissipation, and is found to be in good agreement with previous numerical and experimental data.

3.1.2 Computational Model

We consider only the case of locally-reacting acoustic liners where no communication between liners is possible. Under this assumption, they will have no mutual interaction. Following Tam et al. (2001), the rectangular slit liner measured at Georgia Tech is modeled as being effectively two-dimensional along the mid-span of the slit. It is further hypothesized that the essential energy
conversion occurs away from the ends of the high aspect-ratio slit. The physical model is shown in Fig. 3.2.

### 3.1.3 Grid Size

The computational domain is discretized by a non-uniform grid of size in the $x$ and $y$ direction respectively ($1201 \times 1001$). Near the aperture, the mesh spacing is $0.01D$ in each direction and $101 \times 101$ nodes are included (see Fig. 3.3). Away from the liner opening the grid is coarsened to minimize the total number of grid points. The extension from fine meshing to coarse meshing is given by the tanh function to ensure a smooth transition.

### 3.1.4 Non-Dimensionalization

The simulation used an in-house compressible Navier-Stokes code which was nondimensionalized as:

From these non-dimensional variables, the Reynolds number is defined as:

$$ Re = \frac{\rho_s a_s D^*}{\mu_\infty} $$

Likewise, the Prandtl number is given by

$$ Pr = \frac{C_p^* \mu_\infty^*}{\kappa_\infty} $$

where $C_p^*$ is the specific heat at constant pressure and $\kappa_\infty^*$ is the thermal conductivity. These values were set to

$$ Re = 18758.6 \quad Pr = 0.72 $$

for all of the simulations presented in this Chapter.
### 3.1.5 Governing Equation

This problem is a typical compressible viscous flow problem whose governing equations in non-dimensional form may be written as

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} \rho u_j = 0 \tag{3.1}
\]

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_i u_j + p \delta_{ij} - \tau_{ij}) = 0 \tag{3.2}
\]

\[
\frac{\partial \rho E}{\partial t} + \frac{\partial}{\partial x_j} \left( \{\rho E + p\} u_j + q_j - u_i \tau_{ij} \right) = 0. \tag{3.3}
\]

A Newtonian fluid with Fourier heat conduction model is assumed such that shear stress tensor and heat flux vector, respectively, are

\[
\tau_{ij} = \frac{\mu}{Re} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \frac{\lambda}{Re} \frac{\partial u_k}{\partial x_k} \delta_{ij},
\]

\[
q_i = -\frac{\mu}{Pr Re} \frac{\partial \Theta}{\partial x_i}, \tag{3.4}
\]

where \(\mu(\Theta), \lambda(\Theta)\) and \(k(\Theta)\) functions of temperature only and are given by the power law,

\[
\frac{\mu}{\mu_\infty} = \frac{\kappa}{\kappa_\infty} = \frac{\lambda}{\lambda_\infty} = \left( \frac{\Theta}{\Theta_\infty} \right)^n \quad n = 0.666 \tag{3.5}
\]

### 3.1.6 Plane Wave Implementation

The incident acoustic waves, which enter through the top of the domain (see Fig. 3.1) are characterized by amplitude \(A\) and the angle \(\psi\) they make with vertical +\(y\) axis. It is assumed that the incident wave are of the form of linear, monochromatic plane waves given by

\[
\begin{bmatrix}
\rho^i \\
u^i \\
v^i \\
p^i
\end{bmatrix} = \begin{bmatrix} 1 \\
-\sin \psi \\
-\cos \psi \\
1 \end{bmatrix} A \cos[-\omega(x \sin \psi + y \cos \psi + t)]. \tag{3.6}
\]
At 15°C, $A$ is related to decibel level by

$$A = 10^{\frac{\text{SPL}}{20} - 9.701}.$$  

3.1.7 Boundary Condition

Fig. 3.2 provides the boundary conditions applied in the simulation. Note all the thick solid lines are considered the isothermal no-slip boundaries, the dash lines located on the left and right border of the domain are non-reflecting boundaries. The shadow top region is implemented as the sponge zone, in which the initial condition is applied. The sponge zone also absorbs the reflected waves from the facesheet of the liner.

3.2 Results

3.2.1 Qualitative Presentation of Data

At the liner opening, the vertical velocity is a useful indicator of the resonator response. Because of non-linearity, the oscillations are not simple harmonic functions but are influenced by the aperture dynamics and approach a statistically steady state. Fig. 3.4 shows the vertical velocity fluctuation at the center position of the liner aperture for each incident frequency. The maximum magnitude of the vertical velocity is a function of frequency and is not always small relative to the ambient speed of sound.

Under 150 dB intensity acoustic waves, the excited flow field generates vorticity which may shed from the resonator boundaries, depending on the incident wave frequency. The definition of plane vorticity is

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y},$$

and is shown in Fig. 3.5 for each of the incident frequencies. An examination of the flow field suggests that the vortices are usually shed from the sharp corners, as the boundary layer leaves the
aperture wall. After their generation, they may exist in isolation or merge with other vortices and be convected by the excited flow field far away.

The presence of the shed vorticity represents the useful conversion of acoustic kinetic and potential into kinetic energy bound to the vorticity. While it is possible that the vorticity can scatter back into sound, this process is expected to be relatively weak for the current problem. In Fig. 3.5 we find of a significant amount of shed vorticity for frequencies less than or equal to 4 kHz, and a reduced amount at higher frequencies. It appears that for frequencies near and below the resonant frequency there is sufficient time for a vertical flow through the liner aperture to be established, leading to enhanced vortex shedding. This is also seen in Fig. 3.4, where the maximum vertical velocity is around 20% of the ambient sound speed. At higher frequencies the acoustic period is sufficiently short so that a strong flow is not established, with a corresponding reduction in the shed vorticity.

3.2.2 Time-averaged Viscous Dissipation

Viscous Dissipation in the Aperture Region of the Liner

Using the velocity field history, it is straightforward to calculate the viscous dissipation. The time-averaged dissipation at position \((x, y)\) is given by

\[
\mathcal{D}(x, y) = \frac{1}{T} \int_0^T \tau_{ij} \frac{\partial u_j}{\partial x_i} dt
\]

(3.8)

where \(T\) is the period of oscillation and the integrand is the instantaneous viscous dissipation. Fig. 3.6 shows the time-averaged dissipation rate in the aperture region of the liner. It is seen that as the frequency increases the dissipation moves away the shed shear layers towards the aperture walls. Correspondingly the visual amount of dissipation appears to decrease. That the dissipation at the lower frequencies is mostly off the walls is most easily seen in Fig. 3.6(a) where the shed boundary layer is visible.
Total Viscous Dissipation by the Liner

Tam and Kurbatskii (2000) argued that the total acoustic energy dissipation rate, denoted by $E_{\text{dissipation}}$, is closely related to the mechanisms of the flow field such that it can be classified as

$$E_{\text{dissipation}} \approx E_{\text{viscous}}^A$$ \hspace{1cm} \text{without vortex shedding} \hspace{1cm} (3.9)$$

$$E_{\text{dissipation}} \approx E_{\text{viscous}}^A + E_{\text{shedding}}$$ \hspace{1cm} \text{with vortex shedding,} \hspace{1cm} (3.10)$$

where superscript $A$ denotes the area of the liner aperture region and $E_{\text{viscous}}^A$ is the viscous dissipation in the aperture region of the liner.

By integrating Eqn. 3.8 in the region $A$, the viscous dissipation rate in the aperture region of the liner is found to be

$$E_{\text{viscous}}^A = \iint_A D(x, y) \, dx \, dy. \hspace{1cm} (3.11)$$

To evaluate $E_{\text{shedding}}$, Tam and Kurbatskii (2000) counted the vortices created in unit time $\frac{N}{T}$, treated each vortex carried the same amount of kinetic energy, $K$. Thus, Tam and Kurbatskii (2000) estimated,

$$E_{\text{shedding}} = \frac{KN}{T} \hspace{1cm} (3.12)$$

is the rate of production of kinetic energy bound to vorticity.

In contrast, our way of the evaluation of the total time-averaged acoustic energy dissipation in a period comes directly but needs to meet two requirements. First, it is necessary to set a large enough integral domain (see Fig. 3.7, denoted by $V$) that ensures all the vortices are dissipated within the area. Second, the excited flow should reach a relative steady state.

Following the above explanation, once the oscillation tends to be a steady state, it is assumed that the kinetic energy carried by shed vortices also reaches a steady state. Since no vortices can escape from the region $V$ and they will ultimately be dissipated in their slow migration processes,
the total viscous dissipation rate \( E^V_{viscous} \) can be directly calculated by integrating Eqn. 3.8 over the selected region \( V \) in the whole computational domain (see Fig. 3.7) and is given by,

\[
E^V_{viscous} = \int \int_V \mathcal{D}(x, y) \, dx \, dy \tag{3.13}
\]

Thus, the total energy dissipation rate, regardless of whether there is vortex shedding, can be approximated as,

\[
E_{dissipation} \approx E^V_{viscous} \tag{3.14}
\]

### 3.2.3 Absorption Coefficient Analysis

It is straightforward to evaluate the energy flux of the incident acoustic waves through an area \( S \) to the aperture of the resonator, which is equal to

\[
E_{incident} = \frac{p^2_i S}{\rho_\infty a_\infty} \tag{3.15}
\]

Based on Eqn. 3.15 and the above analysis, the absorption coefficient (fraction of total sound power absorbed) is given by

\[
C_{abs} = \left( \frac{E_{dissipation}}{E_{incident}} \right) \left( \frac{p^2_i}{\rho_\infty a_\infty} \right) A_s = \frac{E_{dissipation}}{E_{incident}} \cdot \frac{A_s}{A_t} \tag{3.16}
\]

where \( A_s \) and \( A_t \) are the area of the liner aperture and area of the resonator, respectively.

Fig. 3.8 shows a comparison of the absorption coefficients measured experimentally and the numerical results calculated by Tam, Kurbatskii, Ahuja, and R. J. Gaeta (2001). As is evident, except for the 6 kHz case, the agreement with the experimental data is reasonable. Moreover, it also shows a continuous decrease in the absorption coefficient with increasing frequency until 5 kHz, after which there appears to be an increase. For \( f \leq 5 \text{ kHz} \) the continual decrease is tied to the decreased amount of vorticity production with increasing frequency, and illustrates the utility of vortex shedding as an acoustic energy reduction mechanism.
Note also in Fig. 3.8 the uncertainty bars are associated with each value of $C_{abs}$. These bars indicate the range of $C_{abs}$ values obtained when different cycles were considered. In general the uncertainty decreases with increasing frequency. Note also that no bar is shown for the 1 kHz result. It is interesting to point out that the discrepancy between the experimental and numerical data in 6 kHz sound wave is found in both Tam et al. (2001) and the present numerical investigations.

### 3.3 Energy Equation Balance Validation

The verification of the energy equations term-by-term is a necessary requirement to check whether the absorption coefficient analysis of the previous section is robust. Using the definitions:

\[
\text{Numerical Error: } \epsilon(x, y) = \frac{\partial \rho E}{\partial t} - \frac{\partial}{\partial x_j} \left((\rho E + p)u_j + q_j - u_i \tau_{ij}\right) \quad (3.17)
\]

\[
\text{Viscous Dissipation: } D(x, y) = \frac{\partial u_i}{\partial t} \tau_{ij} \quad (3.18)
\]

\[
\text{Heat Flux: } Q(x, y) = -\frac{\partial k}{\partial x_j} \frac{\partial \Theta}{\partial x_j} - k \frac{\partial^2 \Theta}{\partial x_j \partial x_j} \quad (3.19)
\]

\[
\text{Ratio: } R \triangleq \frac{\|\epsilon(x, y)\|}{\max(\|D(x, y)\|, \|Q(x, y)\|)} \quad (3.20)
\]

<table>
<thead>
<tr>
<th>Point</th>
<th>Location</th>
<th>Heat Flux</th>
<th>Viscous Dissipation</th>
<th>Numerical Error</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(-0.5, -2.0)</td>
<td>$2.5 \times 10^{-6}$</td>
<td>$8.1 \times 10^{-6}$</td>
<td>$-7.4 \times 10^{-7}$</td>
<td>$9.1 \times 10^{-2}$</td>
</tr>
<tr>
<td>B</td>
<td>(0.0, -2.0)</td>
<td>$-3.5 \times 10^{-6}$</td>
<td>$1.0 \times 10^{-5}$</td>
<td>$-9.2 \times 10^{-7}$</td>
<td>$2.6 \times 10^{-2}$</td>
</tr>
<tr>
<td>C</td>
<td>(0.3, -0.5)</td>
<td>$1.0 \times 10^{-4}$</td>
<td>$5.7 \times 10^{-5}$</td>
<td>$-6.6 \times 10^{-6}$</td>
<td>$6.6 \times 10^{-2}$</td>
</tr>
<tr>
<td>D</td>
<td>(-1.5, -1.5)</td>
<td>$1.1 \times 10^{-5}$</td>
<td>$5.8 \times 10^{-7}$</td>
<td>$8.6 \times 10^{-8}$</td>
<td>$8.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>E</td>
<td>(-0.3, -0.5)</td>
<td>$-1.1 \times 10^{-5}$</td>
<td>$1.5 \times 10^{-7}$</td>
<td>$5.9 \times 10^{-8}$</td>
<td>$4.9 \times 10^{-3}$</td>
</tr>
<tr>
<td>F</td>
<td>(0.0, -3.5)</td>
<td>$-2.6 \times 10^{-7}$</td>
<td>$8.6 \times 10^{-10}$</td>
<td>$-6.0 \times 10^{-9}$</td>
<td>$2.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>G</td>
<td>(-0.5, -2.5)</td>
<td>$1.0 \times 10^{-9}$</td>
<td>$7.9 \times 10^{-10}$</td>
<td>$1.2 \times 10^{-12}$</td>
<td>$9.9 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 3.1: Node-based energy equation balancing.

Table 3.1 indicates that the numerical errors are at least an order of magnitude smaller the viscous dissipation terms. Points A to E are near the aperture region where the magnitude of viscous dissipation is high; Points F and G are where the magnitude of the dissipation rate is low.
It is also noteworthy that the maximum value of $Q$ may exceed the dissipation. A successful analysis of the dissipation rate is closely related to the magnitude of the numerical error, which should not exceed the 10% of the viscous dissipation at the places at viscous dissipation is high. The numerical error of all the points except F are well bounded.

### 3.4 Figures for Chapter 3

Figure 3.1: Schematic of the physical model of the normally incident waves propagating in a quiescent medium.
Figure 3.2: Geometrical model used in numerical simulations. Legend: −, isothermal no-slip boundary; −−, non-reflecting boundary.

Figure 3.3: Mesh details in the upper left corner of the resonator aperture.
Figure 3.4: Vertical velocity at the center point \((0, -0.5)D\) of resonant liner aperture region. Figures (a)-(f) correspond to frequencies 1, 2, \ldots, 6 kHz, respectively.
Figure 3.5: Contours of vorticity by different sound frequencies at 150 dB. Figures (a)-(f) correspond to frequencies 1, 2, . . . , 6 kHz, respectively. Contour levels $\zeta \in [-0.5, 0.5]$. 

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Figure 3.6: Contours of time-averaged viscous dissipation by different sound frequencies at 150 dB. Figures (a)-(f) correspond to frequencies 1, 2, ..., 6 kHz, respectively. Contour levels $\overline{D}(x, y) \in [0, 10^{-4}]$. 
Figure 3.7: Integration domain for dissipation rate analysis.

Figure 3.8: Absorption coefficient as a function of frequency for a 150 dB incident sound wave. ▲ Tam and Kurbatskii (2000) numerical results; ▣ experimental data of Tam et al. (2001); ● present numerical results.
Figure 3.9: Contours of instantaneous viscous dissipation of 150 dB 3 kHz incidence near the acoustic liner aperture region. Contour levels $D(x, y) \in [0, 10^{-6}]$. 
Chapter 4

Grazing Flow Liner Impedance Predictions

4.1 Introduction

In the previous chapter, the results for the normal incident wave with different frequencies were presented, with an emphasis on the energy dissipation mechanisms and absorption coefficient. However, the acoustic liner working mechanisms in real engine ducts are more complicated than the model presented above. First, high speed grazing turbulent flows are usually found in most of the jet engine ducts. Secondly, the noise source are usually found to travel in the direction of the mean flow. Till now, most of the DNS were performed with normal incidence in quiescent media in low Mach number grazing flow, as noted in chapter 1. This chapter will present the simulation results from a DNS of grazing flow with grazing incidence.

4.2 Physical Model

Fig. 4.1 provides a schematic figure of the physical model studied in this chapter. Instead of establishing a 3-D model with turbulent boundary layer, compressible laminar boundary layers are introduced and the computational model is simplified to 2-D. The acoustic liner size presented in Fig. 4.1 are based on one of Jing and Sun (1999)’s experimental setups. The incident source is imposed in the inlet of the duct and the plane wave travels in the direction of the mean flow. To make the simulation feasible and keep well-resolved characters near the liner apertures, a non-uniform grid with 1.56 million points was introduced. The boundary layer thickness ($\delta_{99}$) is 30 mm and within this distance, around 200 points are presented to resolve the complicated flow features.
Table 4.1: Numerical simulation cases in Chapter 4.

<table>
<thead>
<tr>
<th>No.</th>
<th>SPL (dB)</th>
<th>Incident Frequency (Hz)</th>
<th>Free Stream Mach Number ($M_{\infty}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>130</td>
<td>500</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>500</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>500</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>500</td>
<td>0.50</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td>500</td>
<td>0.85</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
<td>1000</td>
<td>0.15</td>
</tr>
<tr>
<td>7</td>
<td>150</td>
<td>1000</td>
<td>0.50</td>
</tr>
<tr>
<td>8</td>
<td>150</td>
<td>1000</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Fig. 4.2 reveals the streamwise velocity profile of a compressible boundary layer for the three Mach numbers considered. Fig. 4.3 provides the pressure field inside the cavity and in the engine duct when planewave is imposed. The studies focused on different Mach number grazing flows and different incident acoustic waves (e.g., frequencies and SPLs) and the set of simulation cases run are listed in Table 4.1.

4.3 Vorticity Generation and Shedding Phenomenon

The studies of vorticity apply the same definition (Eqn. 3.7) given in section 3.3. Fig. 4.4 provides a detailed vorticity generation process of 150 dB incidence with 500 Hz frequency in $M_{\infty} = 0.85$ grazing flow. For the first half cycle when the duct pressure is higher than the cavity, fluid is forced into the cavity through the aperture. Because of the sharp corners of the aperture, two large vortices are generated and followed by a series of random smaller vortices. Half a cycle later, the pressure of the incident sound outside the resonator is at a lower level which causes a reversed outflow from the cavity to the duct through the liner aperture. For sufficient large SPL, the vortices are not confined in the boundaries of the liner and are therefore shed and move away. Because of the presence of the grazing flow in the duct, the shed vortices outside the cavity are carried by the grazing flow and migrated downstream, while the studies of no grazing flow in section 3.3 showed the slow migration of shed vortices away from the aperture. Fig. 4.4 also reveals a full observation of the vortices downstream of the resonator. The present simulation agrees well with Tam, Ju, and
Walker (2008) where the vortices persist over a long distance downstream. Similar observations are found in other cases.

Also note that different vortices migrate downstream with different velocity. Fig. 4.4 reveals that the distance between vortex B and vortex C is increasing as they are migrating downstream. Since the migrating velocity depends mostly on two factors: (1) the grazing flow Mach number; (2) the distance of the vortices away from the duct. In the duct, it is, without doubt, that the streamwise velocity increases with the increase of the distance away from the duct wall and vortices farther away from the duct wall are convected faster.

Fig. 4.5 shows global and near aperture vorticity fields for all of the cases listed in Table 4.1. Note that the vortices are seldom generated for 130 dB incidence, while strong vortex shedding is found for cases at 150 dB. A comparison between the images in Fig. 4.5 reveals that vortex shedding is stronger for the lower frequency incident waves. This observation is consistent with the studies in section 3.3 where lower frequencies incidences were found to be more capable of vortex shedding. It should also be noted from Fig. 4.5 that grazing flow velocity also influences the vortex shedding phenomenon. Instead of two train of vortices generated and shed from the left and right neck boundaries, the vortices are mostly generated by the right neck boundary in high Mach number grazing flow.

In high speed grazing flow, once the vortices are ejected out of the cavity, they are immediately carried away by the mean flow. Therefore, nearly all the vortices are confined in the boundary layer. In lower speed grazing flow the shed vortices have sufficient time to migrate vertically outside the boundary layer before being convected downstream and thus convect farther above the duct wall. These vortices appear to survive for a relatively long time and may impact the flow field of the neighboring cavity, if there is one.
4.4 Mass Flow Rate Distribution

Motivated by the time domain liner model of Hersh, Walker, and Celano (2003), certain time-dependent quantities are examined. The mass flow rate is time-dependent and is given by

\[ \dot{m}(t) = \int_0^D \rho(t)v(t)dx \]  

where the integration extends over the entire liner aperture. Note in Fig. 4.6 the mass flow rate is not only related to the incidence SPL but also has a close relationship with the incidence frequency. After the transition period, time-periodic states are found in all the simulation results. The amplitude of the mass flow rate, however, reveals a large difference for different SPL and frequency incidences. First, the mass flow rate of higher SPL incidence (150 dB) fluctuates more violently than the lower SPL incidence (130 dB). A physical explanation of this phenomenon is that higher SPL incident waves are more capable of exciting the flow field than lower SPL incident waves. Secondly, for the same SPL incidence, lower frequency waves tend to have a larger fluctuation than higher frequency waves. Consistency is found with the results from section 3.2 that lower frequency incident waves create larger vertical velocity fluctuations.

Also note that in the lower SPL cases the mass flow rates are nearly sinusoidal after the transition period while for higher SPL cases, especially those at lower frequencies, the mass flow rates tend to have non-linear behavior and double peaks are found in Fig. 4.6(e) and Fig. 4.6(g).

4.5 Displacement Thickness Distribution

In section 4.3, the vorticity contours revealed a significant boundary layer character in the aperture region. While mass flow rate gives a rough idea of the excited flow field, the displacement thickness distribution along the neck of the acoustic resonator provides a more detailed study of the flow field near the aperture region. Because of the presence of the grazing flow, it is necessary to study the
displacement thickness on both neck walls. By traditional definition of displacement thickness,

\[
\delta_L^* = \int_0^D \left[ 1 - \frac{(\rho(x,t)v(x,t))}{(\rho(t)v(t))_{ref}} \right] \, dx \tag{4.2}
\]

\[
\delta_R^* = \int_0^D \left[ 1 - \frac{(\rho(x,t)v(x,t))}{(\rho(t)v(t))_{ref}} \right] \, dx \tag{4.3}
\]

where \(\delta_L^*\) and \(\delta_R^*\) denote the left and right displacement thickness respectively and \((\rho(t)v(t))_{ref}\) is a reference position, chosen to be at the centerline of the aperture.

Fig. 4.7 reveals that the left and right displacement thickness can be different and this difference is closely related to the free stream Mach number because of the presence of the grazing flow. Also note the displacement thickness distributions, especially the right displacement thickness, behave differently between the first half cycle and the second half cycle. The integral in Eqn. 4.2 and Eqn. 4.3 diverges when \(v_{ref} = 0\) and these singularities form clear boundaries for the half cycles of the flow field. Note in Fig. 4.7(a) and Fig. 4.7(b), for 130 dB incident waves, for the first half cycle when pressure outside the cavity is higher than the pressure inside the cavity the left displacement thickness tends to grow, although a decrease may be found in some small intervals, until the reference velocity goes to zero. No significant difference is found for the left displacement thickness distribution for the second half cycle. However, with the presence of the grazing flow, Fig. 4.7(b) reveals an obvious trough for the right displacement thickness distribution. Both the left and right boundary layer has a nice periodic behavior after several periods.

For 150 dB incident waves, the displacement thickness distribution reveals a different character. Note in Fig. 4.7(c), Fig. 4.7(e) and 4.7(g), the right displacement thickness are near zero except for the flow reversal regions. This phenomenon was not observed in Fig. 4.7(c), 4.7(e) and 4.7(g). Also note in these figures, wiggles emerge in both of the left and right displacement thickness distributions.

Also note in Fig. 4.7(a) that the boundary layer behavior almost collapse when there is no grazing flow in the duct. At this condition, the excited flow field is statistically symmetric and the vortices are being generated at the same amount by these two boundaries. The phase difference between the left and right boundary can be neglected since the aperture size is much smaller than
the wavelength. Because the shed shear layers are Kelvin-Helmholtz unstable and vortex patterns are no longer symmetric (see Fig. 4.5(b)), the left and right displacement thickness may be slightly different but keep with similar profiles.

4.6 Wall Shear Stress Distribution

The wall shear stress along the neck of the aperture is another relevant quantity for time-domain models. It is defined here as

\[
\tau_L = \frac{1}{Re} \int_0^d \mu \frac{\partial v}{\partial x} \left| \text{left wall} \right| dy
\]

and

\[
\tau_R = \frac{1}{Re} \int_0^d \mu \frac{\partial v}{\partial x} \left| \text{right wall} \right| dy
\]

where \(\tau_L\) and \(\tau_R\) denote left and right wall shear stress respectively and \(d\) is the thickness of the perforated sheet plate.

Fig. 4.8 provides the wall shear stress distribution in time domain. It can be directly observed that lower amplitude of the wall shear stress fluctuations are found in 130 dB cases. For all of the cases using 150 dB incident waves, higher amplitudes of the wall shear stress are observed in the lower frequency (500 Hz) cases. Analysis in the frequency domain serves better for the more information of the wall shear stress distribution. By applying the fast Fourier transform (FFT) method, results are presented in Fig. 4.9. A comparison between Fig. 4.9(b) and Fig. 4.9(c) shows lower wall shear stress are found at lower SPL conditions. The physical explanation of which can be traced back to the fact that higher SPL incidence are more capable of exciting strong nonlinear flow field, which undoubtedly cause larger vertical velocity gradient along the aperture boundaries, also consistent with the mass flow results shown earlier. Also note that in Fig. 4.9(d), 4.9(f) and 4.9(h), there is always a difference between the left and right shear stress strength and this difference is increased when the grazing flow Mach number is increased. The physical explanation of this phenomenon can be attributed to the fact that the symmetry is broken due to nonlinear effects becoming dominant as the Mach number of the grazing flow increases. Symmetry of the wall
shear stress is achieved in the case of when there is no grazing flow in the duct. (see Fig. 4.9(a)).

A further inspection of the spectra of wall shear stress reveal two different phenomena. Fig. 4.9(d) shows that for 1000 Hz cases, the spectra has an obvious peak around 1000 Hz. The corresponding resonance is considered as the direct result of the incoming incidence. However, for all 500 Hz cases, it is observed that more than one peak exists in the frequency domain. Moreover, it is interesting that these peaks are located above 1000 Hz. Related to the fact that 500 Hz incidence are more capable of vortex shedding, it is reasonable to explain why these peaks are above 1000 Hz.

4.7 Impedance Prediction

As one of the most significant design parameters, acoustic impedance is defined as the ratio of acoustic pressure to acoustic velocity at a point on the surface of the panel and is given by the complex number,

\[ Z = \frac{p^a}{v^a} = R + iX \quad (4.6) \]

where \( Z \) is the impedance value, \( p^a \) and \( v^a \) are acoustic pressure and acoustic velocity respectively, and \( R \) and \( X \) are acoustic resistance and reactance respectively. The value of impedance that provides the maximum sound absorption at a given frequency depends on the acoustic mode or ray angle of the propagating sound wave.

The normalized impedance by characteristic impedance of air \( \rho a \) is given by

\[ \frac{Z}{\rho a} = z = \theta + i\chi = \frac{R}{\rho a} + i\frac{X}{\rho a} \quad (4.7) \]

where \( z \) is the normalized impedance ratio, \( \theta \) and \( \chi \) are the normalized resistance and reactance ratio respectively.

There are several traditional ways to determine the impedance of acoustic treatment panels experimentally Motsinger and Kraft (1991):

- by measurement of the direct current (dc) flow resistance of the constituents of the the panel
for input to an analytical impedance model

- by measurement of the standing wave pattern in a normal-incidence impedance tube using either a traversing probe or two (or more) fixed pressure transducers
- by measuring the *in situ* impedance with sensors attached to the face sheet and inside the panel cavity.

It should be noted that the first two methods are suitable when grazing flow effects are negligible concern while the last method permits impedance measurement in a duct.

Fig. 4.10 shows the apparatus which is used for determination of impedance by the more specialized in situ method. The method is often called the two-microphone method which was first studied by Dean (1974). Note in the figure one sensor is mounted flush on the backplate of a chosen cavity and the other is inserted through the face sheet. The sensors should be small enough to have negligible effects on the propagation within the cavity. The two-channel spectral analyzer is used to obtained the amplitude and phase of the two pressure signals relative to one another. In this case the normalized impedance for an SDOF is related to

\[
z = -i \frac{p_A}{p_B} \frac{e^{i\phi}}{\sin(kH)}
\]  

(4.8)

Where \(p_A\) and \(p_B\) are pressure at the surface and backplate of the cavity respectively, \(\phi\) is the phase difference between pressure signal \(A\) and \(B\) and \(H\) is the depth of the cavity. Motivated by the idea of the two microphone method, a similar approach can be adopted to process the impedance prediction after the simulation results are available. From the original definition of acoustic impedance in Eqn. 4.6 and the detailed derivation by Dean (1974), the center point of the phase of the acoustic pressure signal is concerned. The presence of the grazing flow, however, makes it hard to isolate the acoustic pressure from the hydrodynamic pressure. Therefore, measurements should be taken at a position where the flow field does not influence the acoustic pressure but not too far away from the center point of the cavity. Most of the previous work of the acoustic liner impedance prediction were concerned with normal incidence and there is no need to consider
the phase difference provided the measurement is in the vicinity of the aperture. In this work, since the incident wave travels along the duct as well as the grazing flow effect, pressure signals at different locations have different phases and should be corrected before calculating the phase different between the pressure of the perforated surface sheet and the pressure of the backplate of the cavity. Fig. 4.11 provides a schematic method of the phase correction at different locations and detailed correction formula is given by

$$\Delta \phi_i = \frac{d_i}{\lambda(1 + M_\infty)},$$  \hspace{1cm} (4.9)

where $d_i$ is the distance between the selected point $A_i$ and the center point $A$ of the aperture and $\Delta \phi_i$ is their phase difference relation. Note in the figure the phase difference between point A and B is needed for the acoustic impedance prediction and their phase difference is given by

$$\phi_{AB} = \phi_{A_B} - \Delta \phi_i$$  \hspace{1cm} (4.10)

To give a complete understanding of the acoustic prediction, four upstream points of the aperture at different locations and the center point of the aperture are selected and are listed in Table 4.7.

<table>
<thead>
<tr>
<th>No.</th>
<th>label</th>
<th>$d_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$A$</td>
<td>0.0</td>
</tr>
<tr>
<td>1</td>
<td>$A_1$</td>
<td>15.9</td>
</tr>
<tr>
<td>2</td>
<td>$A_2$</td>
<td>21.3</td>
</tr>
<tr>
<td>3</td>
<td>$A_3$</td>
<td>30.0</td>
</tr>
<tr>
<td>4</td>
<td>$A_4$</td>
<td>42.5</td>
</tr>
</tbody>
</table>

Table 4.2: Measurement points at different locations upstream of the aperture.

Fig. 4.12 presents the acoustic impedance prediction under low SPL (130 dB) of the simulation results and experimental data. Note that when there is no grazing flow ($M_\infty = 0.0$), excellent agreements of the resistance are found in the value of point $A$, $A_1$ and $A_2$ prediction. The resistance prediction of the point $A_3$ and $A_4$ has some discrepancy but is still in the acceptable range. Since all of the values are very close to zero, which means the backplate pressure and surface pressure
are out of phase, a little phase error will cause a large difference. Reasonable agreement are found in reactance value predictions.

Under high SPL (150 dB), the acoustic impedance predictions are shown in Fig. 4.13. As there are no experimental data, comparisons are made with the empirical formula and the theoretical formula by Rice and Edward (1971) and Rice (1976), respectively. According to Motsinger and Kraft (1991), the empirical data is only an acceptable estimate of the resistance prediction when $M_\infty \in [0.3, 0.4]$.

As shown in both Fig. 4.13(a) and Fig. 4.13(c), the numerical resistance value prediction linearly increases as the increase of $M_\infty$, which has the same trend with both of the empirical prediction and theoretical prediction. Moreover, Fig. 4.13(c) reveals a good agreement of the resistance value predictions with the theoretical formula for 1000 Hz cases. Fig. 4.13(b) reveals a reasonable agreement with the numerical reactance value prediction with the empirical formula for 500 Hz cases.

Also note that after a comparison between Fig. 4.13(b) and Fig. 4.13(d), it is found that both numerical predictions of the acoustic resistance and resistance have frequency dependencies, which was not shown in the empirical formula in Rice and Edward (1971) and in the theoretical expression Rice (1976). Correspondingly, Fig. 4.13(a) reveal the discrepancies between the numerical prediction of the reactance value and the theoretical data for 1000 Hz are found. But it should be noted the high SPL might be a potential nonlinear factor to violate the linear assumptions required in the theoretical formula.

### 4.8 Figures for Chapter 4
Figure 4.1: Schematic of plane wave propagation and detailed size of the acoustic liner computational model.

Figure 4.2: Streamwise velocity profile within the compressible boundary layer. Legend: $- M_\infty = 0.15$; $-- M_\infty = 0.50$; $-\cdot- M_\infty = 0.85$. 

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Figure 4.3: Pressure contours of 150 dB incident planewave imposed in the duct. \((M_\infty = 0.85)\).
(a) $t = 0$

(b) $t = 0.2T$

(c) $t = 0.4T$
Figure 4.4: Vorticity contours of 150 dB incidence at 500 Hz in $M_\infty = 0.85$ grazing flow.
(a) 130 dB 500 Hz $M_{\infty} = 0.0$

(b) 130 dB 500 Hz $M_{\infty} = 0.0$

(c) 130 dB 500 Hz $M_{\infty} = 0.15$

(d) 130 dB 500 Hz $M_{\infty} = 0.15$

(e) 150 dB 500 Hz $M_{\infty} = 0.15$

(f) 150 dB 500 Hz $M_{\infty} = 0.15$

(g) 150 dB 500 Hz $M_{\infty} = 0.50$

(h) 150 dB 500 Hz $M_{\infty} = 0.50$
(i) 150 dB 500 Hz $M_\infty = 0.85$

(j) 150 dB 500 Hz $M_\infty = 0.85$

(k) 150 dB 1000 Hz $M_\infty = 0.15$

(l) 150 dB 1000 Hz $M_\infty = 0.15$

(m) 150 dB 1000 Hz $M_\infty = 0.50$

(n) 150 dB 1000 Hz $M_\infty = 0.50$

(o) 150 dB 1000 Hz $M_\infty = 0.85$

(p) 150 dB 1000 Hz $M_\infty = 0.85$

Figure 4.5: Vorticity contours of the numerical simulation cases listed in Table 4.1.
Figure 4.6: Mass flow rate of the numerical simulation cases listed in Table 4.1.
(a) 130 dB 500 Hz $M_{\infty} = 0.0$

(b) 130 dB 500 Hz $M_{\infty} = 0.15$
(c) 150 dB 500 Hz $M_\infty = 0.15$

(d) 150 dB 500 Hz $M_\infty = 0.15$
(e) 150 dB 500 Hz $M_\infty = 0.50$

(f) 150 dB 1000 Hz $M_\infty = 0.50$
Figure 4.7: Present simulation results of displacement thickness distribution. Legend: – left wall displacement thickness distribution; — — right wall displacement thickness distribution

(g) 150 dB 500 Hz $M_\infty = 0.85$

(h) 150 dB 1000 Hz $M_\infty = 0.85$
Figure 4.8: Present simulation results of wall shear stress: – left wall shear stress; — right wall shear stress
(a) 130 dB 500 Hz $M_{\infty} = 0.0$

(b) 130 dB 500 Hz $M_{\infty} = 0.15$
(c) 150 dB 500 Hz $M_\infty = 0.15$

(d) 150 dB 500 Hz $M_\infty = 0.50$

(e) 150 dB 500 Hz $M_\infty = 0.85$
Figure 4.9: Left column: FFT of left wall shear stress. Right column: FFT of right wall shear stress.
Figure 4.10: Schematic of measurement of grazing flow impedance by two-microphone method.

Figure 4.11: Schematic of phase correction method.
Figure 4.12: Acoustic impedance prediction of 130 dB 500 Hz incidence. Left: acoustic resistance prediction; Right: acoustic reactance prediction. — Empirical model of Rice and Edward (1971); —— Theoretical model of Rice (1976); ⊕ experimental data of Jing, Sun, Wu, and Meng (2001); • Present prediction based on point $A_1$; ♦ Present prediction based on point $A_2$; ▲ Present prediction based on point $A_3$; ⋆ Present prediction based on point $A_4$. 
Figure 4.13: Acoustic impedance prediction of 150 dB incidence: Left: acoustic resistance prediction; Right: acoustic reactance prediction. — empirical model of Rice and Edward (1971); — the theoretical model of Rice (1976); • Present prediction based on point $A_1$; ♦ Present prediction based on point $A_2$; ▲ Present prediction based on point $A_3$; ⋆ Present prediction based on point $A_4$. 
Chapter 5

Conclusions

5.1 Present Work

The purpose of the work presented above was to apply the predictive numerical approach in acoustic liner eduction, using two typical cases as documented above. From these two different studies of acoustic liners under different conditions, the following conclusions are drawn:

1. For acoustic liners in a quiescent medium
   - The shedding of microvortices is an important mechanism to dissipate acoustic energy. However, the capability of vortex shedding is closely dependent on the frequency of the incident sound wave. Usually, incident waves with lower frequencies than the resonant frequency are more efficient in vortex shedding.
   - Because of the existence of the molecular viscosity, the kinetic energy carried by these microvortices will be ultimately converted into heat. The total acoustic energy has two parts, the viscous dissipation by the acoustic liner aperture and kinetic energy carried by the shed vortices. It was revealed that the vortex shedding is a more efficient approach in acoustic energy dissipation than pure viscous dissipation by the liner aperture.
   - To quantify the acoustic energy dissipation, the absorption coefficient was applied. Based on the viewpoint that the kinetic energy carried by these vorticies will be ultimately dissipated by viscosity, we directly integrate the dissipation over a sufficient large domain that almost no vorticies can be convected outside the integral domain. This method is more direct and easy to understand, and shows reasonable agreement with experimental data.
2. For acoustic liners in grazing flow

- Strong vortex shedding was found in high SPL incident waves. At the same SPL, incident waves with lower frequency will create stronger vortex shedding. For high Mach number grazing flow, the shed vortices are mostly bound to the boundary layer and will be dissipated in a fast manner. For low Mach number grazing flow, the shed vortices are found outside the boundary layer and can survive the process of being convected downstream. These vortices will probably interfere the neighboring liners.

- Boundary layer behavior along the acoustic liner neck was studied through the evaluation of the displacement thickness and was revealed that different distribution existed between the left and right boundary. The difference is caused by the grazing flow and increases when the Mach number of the grazing flow increases.

- The wall shear stress distribution have a kinematic consistency with the vortex shedding phenomena: Higher amplitude fluctuations of the wall shear stress corresponds to strong vortex shedding. The difference between the left and right wall shear stress increases with the increasing of grazing flow Mach number.

- The impedance was evaluated through the viewpoint of the traditional two-microphone method and reasonable agreement was found in the resistance prediction. However, discrepancies were found in the impedance predictions for high SPL incident waves.

5.2 Future Work

The work presented above has several limitations, which directs the road of future work. First, the 2-D simulation model is limited in its ability to model the real simulation of the working mechanism for the acoustic liner, as most of grazing flows in the engine are found to be compressible and turbulent. A 3-D computational model with LES is required for these future simulation.

Secondly, the multi-influence of the neighboring liners was reported but few investigations have been made. These investigations would be greatly helpful to the aperture spacing designs.
Thirdly, alternate geometry and configurations of the acoustic liners may be preferred. It is possible that a numerical approach can be a useful tool to provide optimal designs for acoustic liners.


